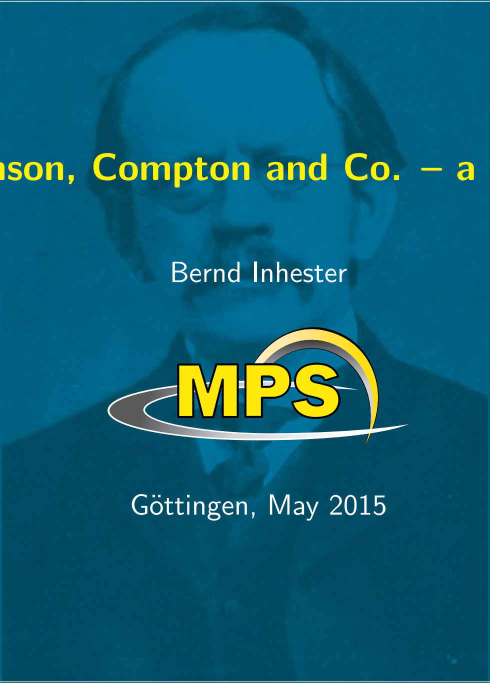
Thomson Scattering Revisited

Bernd Inhester

Göttingen, May 2015



Thomson, Compton and Co. – a Review

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A brief history of Thomson scatter

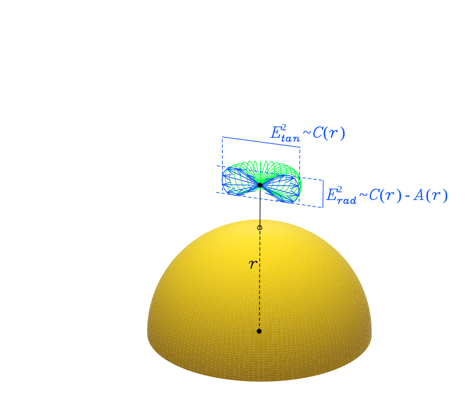
- >1850 Fresnel, Faraday and others demonstrate polarisation of light and its modification by a magnetic field
- 1860 Secchi and Pražmowski independently made first observations of the polarisation of the coronal light during an eclipse.
- 1864 Maxwell summarises his studies on the wave-like nature of light in "A Dynamical Theory of the Electromagnetic Field".
- 1871 Rayleigh calculates the scattering cross section of small polarisable spheres to explain colour and polarisation of light from the Earth's atmosphere.
- 1879 Schuster estimates the coronal brightness and polarisation from scattering using Rayleigh's cross section.

Schuster (1879)

He knew: light is a transversely polarised electromagnetic wave

He calculated: field fluctuations at distance r from an extended Sun

$$E^2_{\text{tan}} \propto C(r) \propto r^{-2}$$
$$E^2_{\text{rad}} \propto C(r) - A(r) \propto r^{-4}$$



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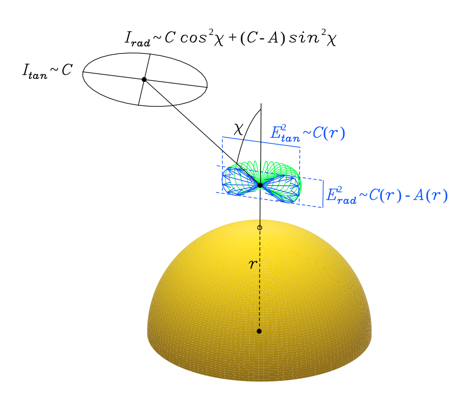

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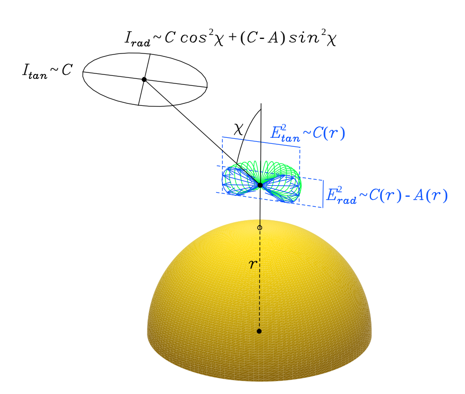


He adopted: Small dipole scattering from Rayleigh → scattered field ∝ projection of field fluctuations transverse to the scattering direction.

- Incident photon direction matters only indirectly
- the electron was not known yet

A brief history of Thomson scatter – continued

...

1879 Schuster estimates the coronal brightness and polarisation from scattering using Rayleigh's cross section.

1896 Thomson proposed existence of electrons from cathode ray experiments

1906 Schwarzschild suggests that the corona is an electron gas considering the photon pressure on different particles.

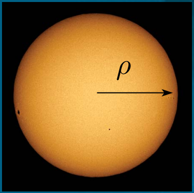
1907 Thomson described the scattering of electromagnetic waves by electrons in 'The Corpuscular Theory of Matter'

1923 Compton extends Thomson scattering to relativistic photon energies.

1930 Minnaert extends Schuster's calculations including limb darkening.

Minnaert (1930)

Minnaert(1930) added the correct Thomson cross section and limb darkening to Schuster's results (two more coefficients):



$B_\odot$ = radiance of Sun centre

$u = 0.58 \ldots 0.63$

$$B(\rho) = B_\odot(1 - u + u\sqrt{1-\rho^2})$$

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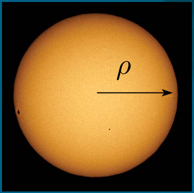

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$$C_{\text{tan}} = \frac{A_{\text{apt}} A_{\text{pix}}}{4\pi f^2} \int_{\text{LOS}} ds \; N_e(\boldsymbol{r}) \frac{r_e^2}{2} \qquad B_\odot((1-u)C(r) + uD(r))$$

$$C_{\text{tan}} - C_{\text{rad}} = \qquad \ldots \qquad N_e(\boldsymbol{r}) \frac{r_e^2}{2} \sin\chi^2 \; B_\odot((1-u)A(r) + uB(r))$$

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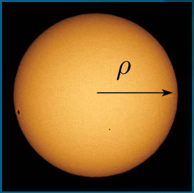

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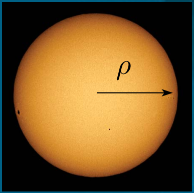

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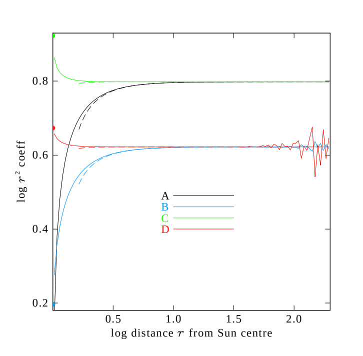
$$C_{\text{tan}} - C_{\text{rad}} = \quad \ldots \quad \underbrace{N_e(\boldsymbol{r})\frac{r_e^2}{2}\sin\chi^2\; B_\odot((1-u)A(r) + uB(r))}_{\text{scattering emissivity } \epsilon(r,\sin^2\chi)\ [\text{photons/s/sr}]}$$

has been used ever since to relate N_e and coronagraph pixel counts

Numerical stability of Minnaert's coefficients

The coefficients A, B, C, D are analytic expressions

For the correct asymtotic decrease $\propto r^{-2}$ large terms have to almost cancel $\rightarrow$ large numerical errors for large r



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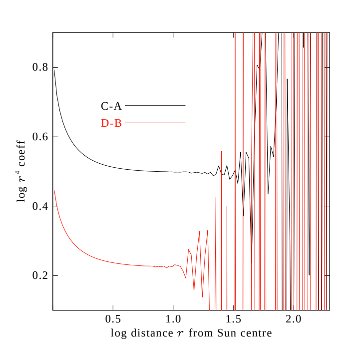


even worse for the combinations needed for the radial polarisation which have to decrase as r^{-4}

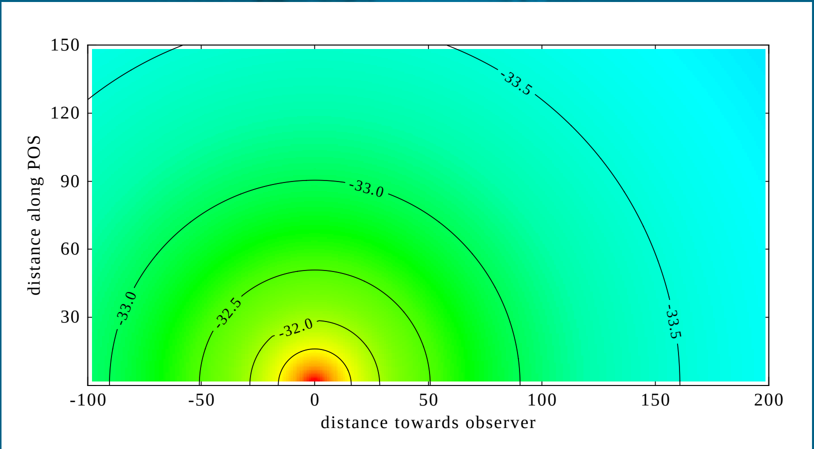
Use asymptotic expansions for $r > 10R_\odot$

Scattering emissivities in the scattering plane

Tangential polarisation ϵ_{tan} (normal to scattering plane):

No dependence on scattering angle χ

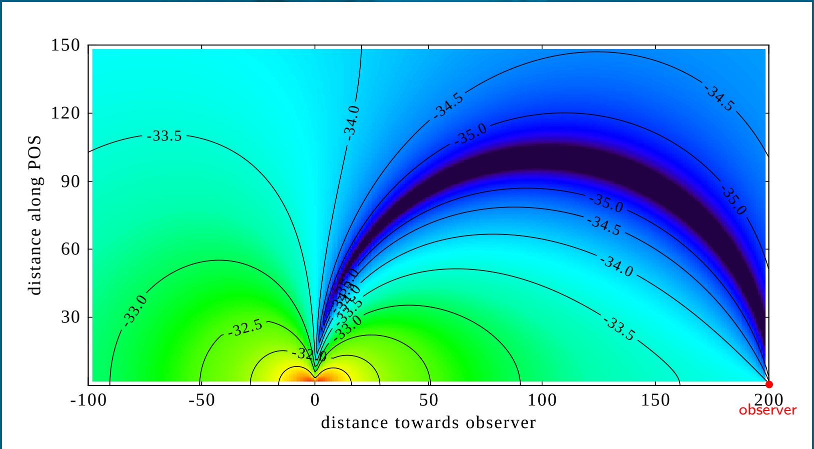
$\rightarrow$ perfectly circular emissivity contours



Scattering emissivities in the scattering plane

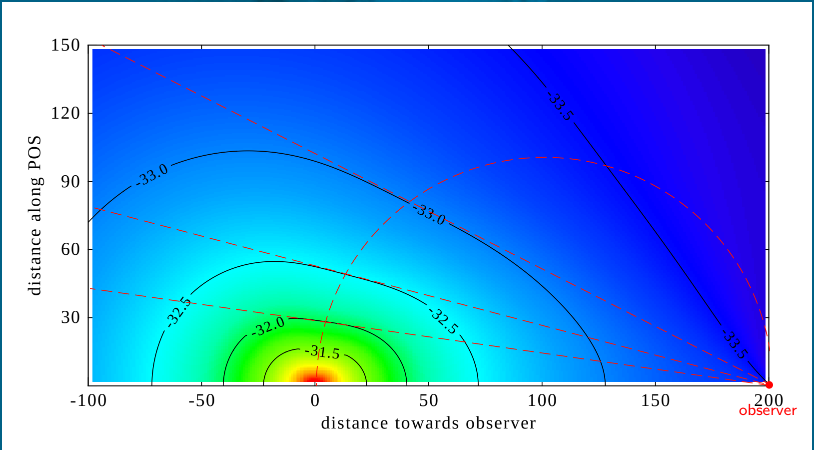
Radial polarisation ϵ_{rad} (polarised in the scattering plane):

No scattering in $\chi = \pi/2$. Smaller than ϵ_{rad} except for scattering in forw/backw direction



Total emmissivity $\epsilon_{\text{tan}} + \epsilon_{\text{rad}}$ in the scattering plane

Dominated by ϵ_{tan}, depleted at $\chi = \pi/2$ due to absence of ϵ_{rad}



Total emmissivity $\epsilon_{\text{tan}} + \epsilon_{\text{rad}}$ in the scattering plane

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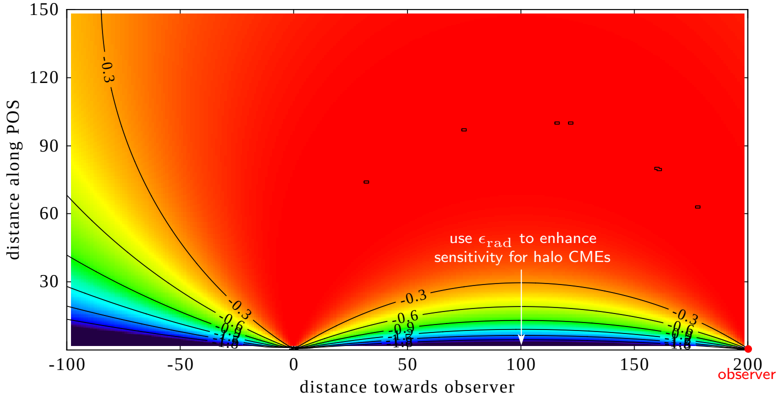
Huge "Thomson sphere" if ϵ is normalised on each ray independently

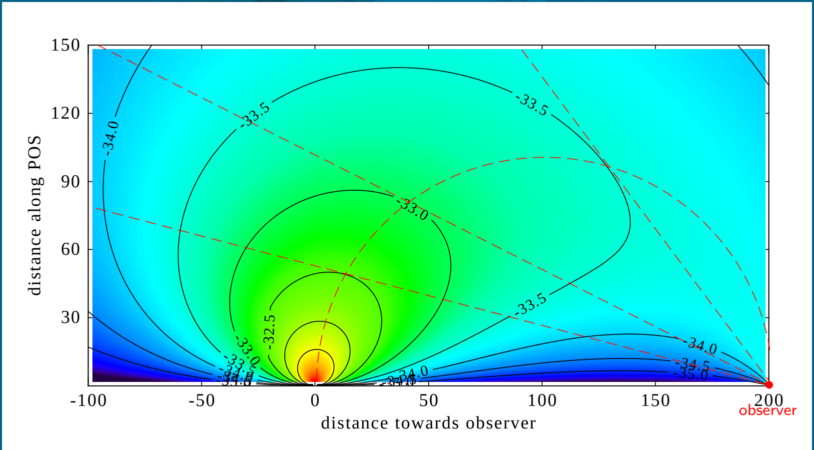
Forw/backw scattering gaps from normalisation by near-Sun maxima



Polarised emmissivity $\epsilon_{\text{tan}} - \epsilon_{\text{rad}}$ in the scattering plane

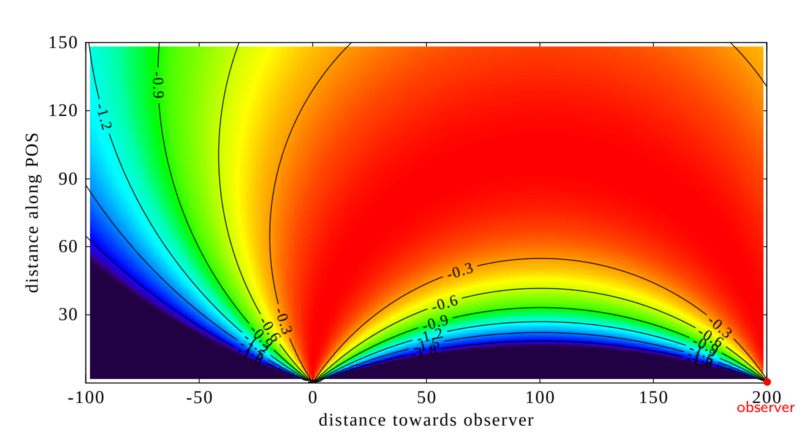
Dominated by ϵ_{tan}, especially at $\chi = \pi/2$ where ϵ_{rad} vanishes

No polarisation in forw/backw direction due to symmetry

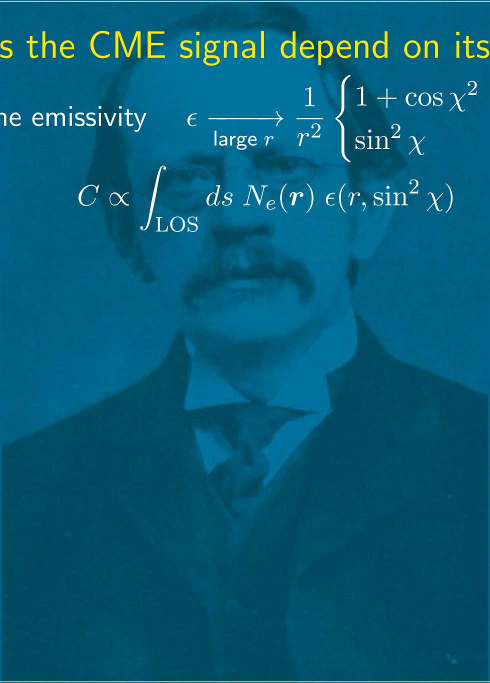


Polarised emmissivity $\epsilon_{\rm tan} - \epsilon_{\rm rad}$ in the scattering plane

- Dominated by $\epsilon_{\rm tan}$, especially at $\chi = \pi/2$ where $\epsilon_{\rm rad}$ vanishes
- No polarisation in forw/backw direction due to symmetry
- "Thomson sphere" more narrowly confined



How does the CME signal depend on its prop dir ?



recall that the emissivity $\epsilon \xrightarrow[\text{large } r]{} \frac{1}{r^2} \begin{cases} 1 + \cos\chi^2 & \text{total B} \\ \sin^2\chi & \text{polarised B} \end{cases}$

and $C \propto \int_{\mathrm{LOS}} ds\, N_e(\boldsymbol{r})\, \epsilon(r, \sin^2\chi)$

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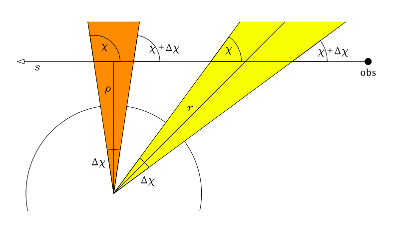

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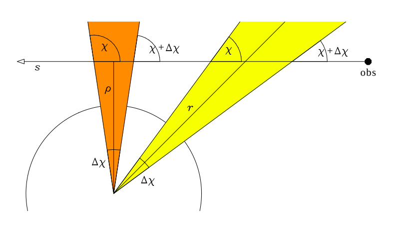

$\boldsymbol{r}$ and $\sin\chi$ depend on s

change integr variable $s \to \chi'$

map range $s = s_{\mathrm{obs}} \ldots \infty$

to $\chi' = \varepsilon \ldots \pi$

use $\frac{ds}{r^2} = \frac{d\chi'}{2\rho}$

yields $$C \to \frac{1}{2\rho} \int_{\varepsilon}^{\pi} N_e(\boldsymbol{r}) \begin{Bmatrix} 1+\cos^2\chi' \\ \sin^2\chi' \end{Bmatrix} d\chi'$$

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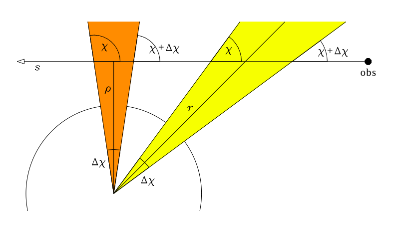

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use $\frac{ds}{r^2} = \frac{d\chi'}{2\rho}$

restrict integration to the CME cones $$C \rightarrow \frac{1}{2\rho} \int_{\chi}^{\chi+\Delta\chi} N_e(\boldsymbol{r}) \begin{Bmatrix} 1+\cos^2\chi' \\ \sin^2\chi' \end{Bmatrix} d\chi'$$

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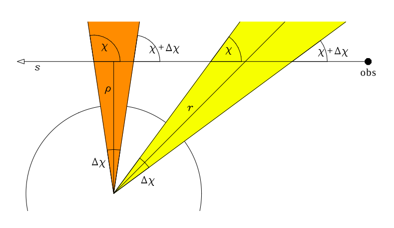

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use $\frac{ds}{r^2} = \frac{d\chi'}{2\rho}$

assume N_e =const $$C \to \frac{1}{2\rho}\, N_e \int_{\chi}^{\chi+\Delta\chi} \begin{Bmatrix} 1+\cos^2\chi' \\ \sin^2\chi' \end{Bmatrix} d\chi'$$

Longer intersection with LOS compensates $1/r^2$ decrease of incident light.
(Howard, DeForest, Tappin, 2009, 2012, 2013; Xiong et al., 2012, 2013)

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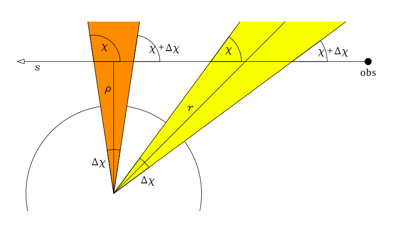

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use $\frac{ds}{r^2} = \frac{d\chi'}{2\rho}$

assume $N_e = A_e \frac{1}{r^2} = A_e \frac{\sin^2\chi'}{\rho^2}$ $\quad C \to \frac{1}{2\rho^3} A_e \int_{\chi}^{\chi+\Delta\chi} \begin{Bmatrix} 1-\cos^4\chi' \\ \sin^4\chi' \end{Bmatrix} d\chi'$

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(Howard, DeForest, Tappin, 2009, 2012, 2013; Xiong et al., 2012, 2013)

A brief history of Thomson scatter – continued

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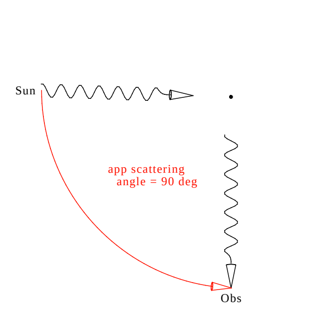
- 1930 Minnaert extends Schuster's calculations including limb darkening.
- 1958 Bowles verifies Thomson scattering by scattering of radio waves from the ionosphere.
- 1960 Salpeter, Fejer, Hagfors and others independently explain the spectral features in Bowles's experiment by "incoherent Thomson scatter"
- 1964 Ramsden & Davies perform first lab experiments on Thomson scattering of laser light.
- 1972 Molodensky points out that frequency and polarisation of coronal brightness may be modified by suprathermal electrons

Limits of plain Thomson scattering

- Low photon energy and/or high electron density:
  Collective plasma response rather than individual particles (Incoherent scatter). No change in overall cross section but spectrum from Gaussian of width $v_{Te} \longrightarrow$ ion lines shifted by $\pm v_{Ti}$ and electron peak at $\pm\omega_{pe}/c$.
  Occurs at $\lambda/\sin(\chi/2) \gtrsim \lambda_{\text{Debye}} = \sqrt{\frac{\epsilon_0 k_B T_e}{e^2 N_e}} = 2\ \ldots\ 7\text{cm}$
- High photon energy:
  Compton recoil of electrons leads to red-shift. Cross section (Klein-Nishida) reduced and concentrated to forw scattering.
  Occurs at $\lambda \lesssim \lambda_{\text{Compton}} = \frac{h}{m_e c} = 2.4\ 10^{-3}$ nm
- High electron energy:
  Still Thomson but in the rest frame of the electron.
  Relativistic transformations of the photon before and after scattering into/from the electron rest frame (inverse Compton scattering) make a change (Molodenski, 1972)
  Occurs at $v_e/c \gtrsim 10^{-1}$, thermal $v_e/c = 1.4\ 10^{-2}$

Inverse Compton: modified effective scattering angle

Electron at rest: polarisation in the scattering plane is not transmitted to observer



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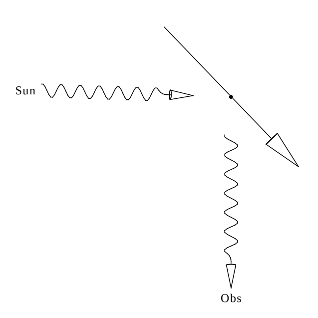

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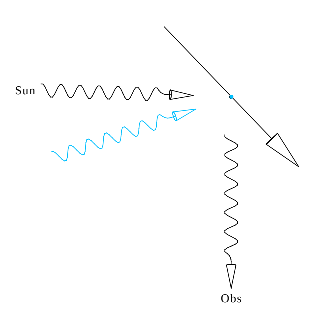

In the electron rest frame:
electron sees incident photon from an aberrated direction

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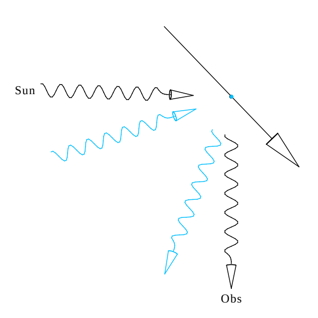

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scattered photon to observer must take account of observer's aberration

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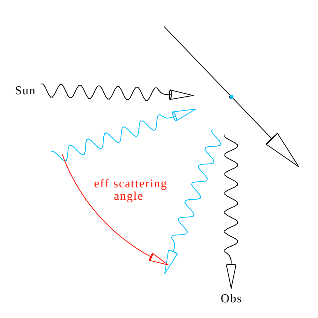
Electron in (relativistic) motion: polarisation in the scattering plane partially transmitted



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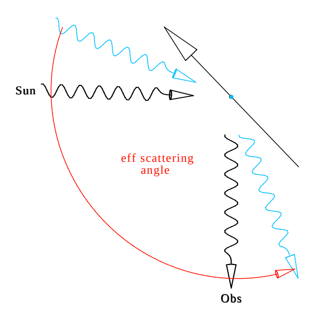


In the electron rest frame: electron sees incident photon from an aberrated direction

scattered photon to observer must take account of observer's aberration

Similar effect if electron velocity is reversed

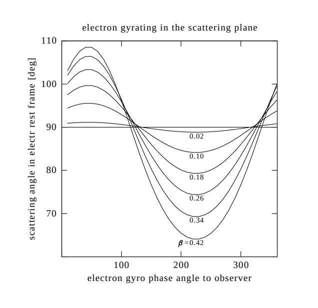
Inverse Compton: loss of polarisation

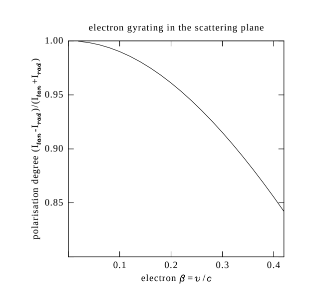
Averaging over all velocities directions of an electron gyrating in the scattering plane results in a decrease of the polarisation:

Radially polarised emissivity changes

from $\epsilon_{\rm rad} \propto \sin^2\chi$

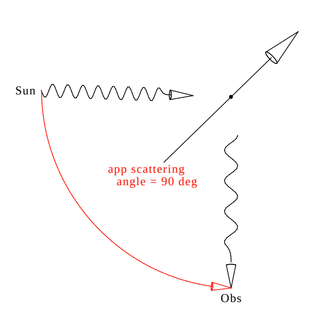
to $\epsilon_{\rm rad} \propto a + (1-a)\sin^2\chi$





Inverse Compton: net frequency shift

Electron velocity bisecting the scattering angle:
no modification of effective scattering angle but a frequency shift



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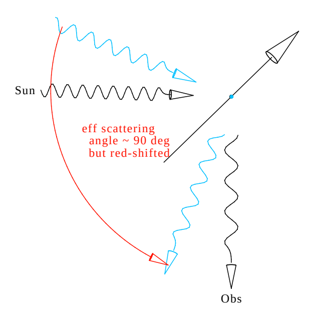


Aberrations to and from the rest frame compensate
→ polarisation preserved

but each transformation yields a frequency shift of the same sign

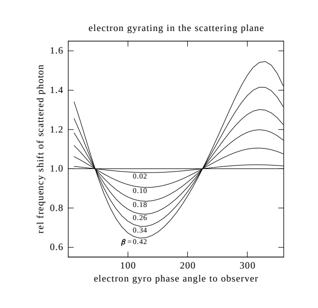
Inverse Compton: the net frequency shift

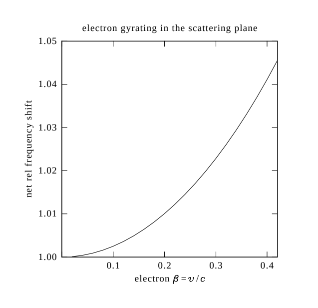
Averaging over all velocity directions of an electron gyrating in the scattering plane results in a net blue-shift:

Frequency shift for each transformation into/out of the rest frame

$$\omega' = \omega \frac{1 + \beta\cos\theta}{\sqrt{1-\beta^2}}$$

gives net blue-shift

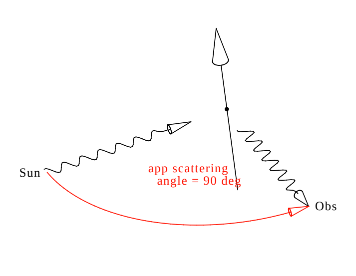




Frequency shift even bigger for electron streams towards/away from Sun (see Kouchmy&Nikoghossian, 2001, 2002, 2005)

Inverse Compton: tilt of polarisation angle

Electron velocity normal to the scattering plane:
Aberration in and out of the rest frame leads to a tilted scattering plane in electron rest frame



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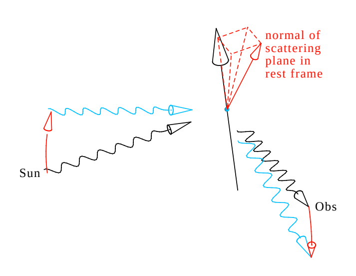

Aberration now bends incoming photon out of the scattering plane. Similar for outcoming photon

Scattering plane in electron rest frame is tilted

Observer sees "tangential" polarisation along projection of scattering normal of rest frame, scattering angle ↑, polarisation ↓

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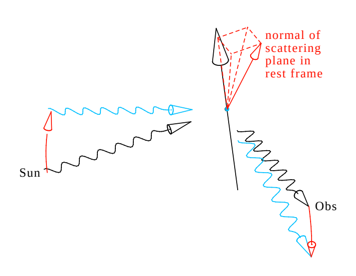

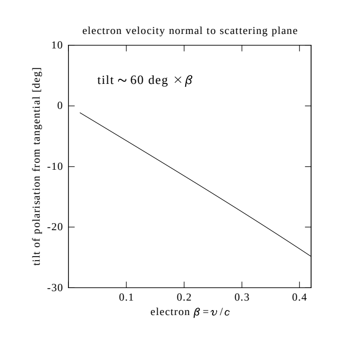

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scattering angle ↑, polarisation ↓

(more details: Molodenski, 1972; Nikoghossian, Koutchmy, 2001, 2005)

Inverse Compton: tilt of polarisation angle

Electron velocity normal to the scattering plane:
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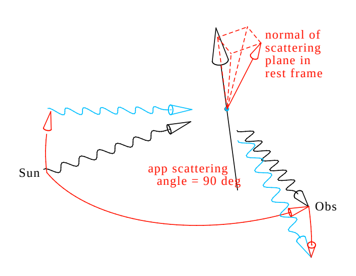

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Summary

- Thomson scattering diff cross section is the simplest cross section possible, phase function is due to Rayleigh.
- The "Thomson sphere", i.e. that scattering is most intense at $\chi = \pi/2$, is due to a number of geometric constraints to which Thomson scattering contributes least:
  - $\oplus$ $\epsilon_{\rm tan} - \epsilon_{\rm rad} \rightarrow \sin^2\chi$
  - $\oplus$ incident light $\propto 1/r^2$
  - $\oplus$ N_e drops with r
  - $\ominus$ $\epsilon_{\rm tan} + \epsilon_{\rm rad} \rightarrow 1 + \cos^2\chi$
  - $\ominus$ longer LOS for through CME cones at $\sin\chi < 1$

$\Rightarrow$ better call it Pythagorean sphere.

- Suprathermal electrons may modify polarisation and frequency. Few observations have been reported on these effects so far – keep a critical eye on the polarisation near active regions, in presence of flares of type III radio bursts.